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SHORT COMMUNICATIONS

Development of thermodynamic relation for mean density of mixture in the risers of natural circulation boilers

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ABSTRACT

Boiler is a closed vessel that is utilized for the purpose of heating liquid, typically water, or for the generation of vapour, steam, or any combination of these substances under pressure for the purpose of external usage through the combustion of fossil fuels. In this article, a development of the mathematical expression for the mean average mixture density of water in the riser of a subcritical natural circulation boiler is presented. Though this expression is presented in several books and literature but the detailed explanation of how the expression comes from is missing. Therefore, this article is an attempt to bridge that gap.

Keywords: Natural circulation boiler, Power Plant, Mean density in riser, Mathematical derivation, Energy efficiency

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ΔP	pressure difference
g	acceleration due to gravity
H	riser height
ρ_m	mean density of mixture in the riser
ρ_D	density of saturated water in downcomer
ρ_T	density of mixture at the exit of riser i.e. at the top
ρ_B	density of mixture at the entry of riser i.e. at the bottom
ρ_f	density of saturated liquid
ρ_g	density of saturated vapour
v_f	specific volume of saturated liquid
v_g	specific volume of saturated vapour
x_o	top dryness fraction
v_g	specific volume of saturated vapour
CR	circulation ratio
$\dot{m}$	total mass flow rate of mixture
$\dot{m}_g$	mass flow rate of vapour
$\dot{m}_f$	mass flow rate of liquid
α	void fraction

$\forall$	volume of mixture
$\forall_g$	volume occupied by vapour
$\forall_f$	volume occupied by liquid
A_g	area occupied by vapour
A_f	area occupied by liquid
S	slip ratio
V_g	velocity of vapour
V_f	velocity of liquid
x	dryness fraction
φ	constant which is having a value $S \times \frac{v_f}{v_g}$
ρ_g	density of saturated vapour
ρ_f	density of saturated liquid
k	constant
TDF or x_o	top dryness fraction
ℓ	length along riser
h	variable height along the riser

1. Introduction

Boiler is considered as very important device for the formation of steam and used for many applications.[1]. These boilers, which are also called steam generators are not merely boiler drums indeed the term boiler is given to a closed circuit comprised of drum, downcomer, bottom ring header, and riser as shown in **Figure 1**[2].

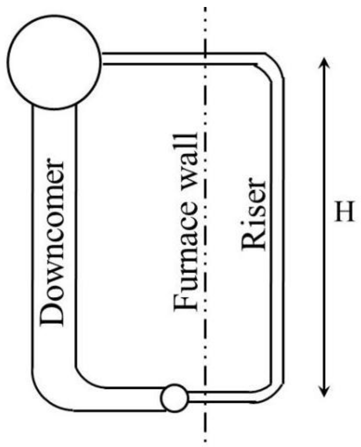

Figure 1. Riser Downcomer circuit [2]

Water falls due to gravity in the downcomer and gets distributed via bottom ring header into small cross section riser tubes. These riser tubes are the ones which act a wall of water hence, called as water walls. As these are inside the furnace, they receive immensely high temperature flue gas due to which the phase conversion from liquid to vapour takes place. This mixture of water and steam moves to the drum due to the lower density of the mixture in riser compared to the downcomer. In the boiler the segregation of steam and water occurs, and this natural circulation cycle continues[2]. Understanding the mean density of the fluid mixture in risers is very important for optimizing the working of natural circulation boilers. The accurate evaluation of mean density directly influences the estimation of pressure drop and circulation rates, which are essential for the safe and efficient operation of the power plants. Errors in estimating the mean density can lead to incorrect circulation predictions which can potentially cause issues like nonuniformity in heat transfer, thermal stresses, or even the failure of a boiler[3].

Circulation is the terminology used in the boiler circuit to represent the flow of water and steam in it. As, in the case discussed above, the density difference is the sole cause of circulation, that's why these types of boilers are also called as natural circulation boilers[4]. The pressure difference responsible for the natural circulation is proportional to the product of riser height and the difference in the densities of fluid in downcomer and riser as shown in Eqn. (1).

$$\Delta P = (\rho_D - \rho_m)gH \quad (1)$$

where, ρ_D is the density of saturated water in downcomer and $\rho_m$is the collective mean density of water and steam in the riser. ρ_D can be easily obtained by using a steam table but the difficulty lies in evaluating ρ_m. In simplify the calculation process, the practitioner takes the average density of fluid at the bottom (ρ_B) and top (ρ_T) of the riser (as shown by Eq. 2). However, this method of calculation is not entirely an accurate approach[2].

$$\rho_m = \frac{\rho_T + \rho_B}{2} \quad (2)$$

where, $\rho_B = \rho_D$ and $\rho_T = \frac{1}{v_f + x_o(v_g - v_f)}$. x_o is the dryness fraction at the top of riser and v_f and v_g are the specific volume of saturated liquid and vapour corresponding to the boiler pressure. While the simple calculation (Eqn. (2)) offers a quick approximation, it often fails to capture the dynamic behavior of the water-steam mixture under changing load and pressure conditions. This limitation can lead to a conservative or inaccurate boiler design thus reducing operational flexibility or efficiency[5]. But the exact expression for evaluating the mean mixture density in the riser is difficult to understand in terms of its origin and development [2].

Therefore, the objective of this article is to derive the expression of mean density of fluid in the riser by explaining each aspect of the steps involved.

2. Few definitions and intermediate development

Circulation ratio (CR) is ratio of the mass flow rate of water in the downcomer to that of mass flow rate of steam leaving the drum [2,6].

$$CR = \frac{\dot{m}}{\dot{m}_g} = \frac{1}{\dot{m}_g/\dot{m}} = \frac{1}{x_o} = \frac{1}{TDF} \quad (3)$$

Void fraction (α) is the ratio of volume occupied by the vapor to that of the total volume [2,6].

$$\alpha = \frac{\forall_g}{\forall_f + \forall_g} \quad (4)$$

If A_g is the area (volume per unit length) occupied by vapors at any cross-section and A_f is the area (volume per unit length) occupied by liquid (**Figure 2**) at the same cross section of the riser, then Eqn. (4) can be further developed as:

$$\alpha = \frac{\forall_g/l}{\forall_f/l + \forall_g/l} = \frac{A_g}{A_f + A_g} \quad (5)$$

or,

$$1 - \alpha = \frac{A_f}{A_f + A_g} \quad (6)$$

$$\frac{\alpha}{1-\alpha} = \frac{A_g}{A_f} \quad (7)$$

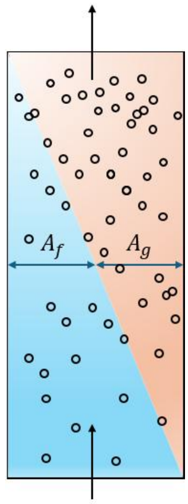

Figure 2. Typical flow of steam and water (two phase flow) in riser.

Slip ratio (S) is the ratio between the velocity of vapour to that of liquid in the riser which is mathematically represented in Eqn. (8). S varies from 1 to 10 (experimentally). As the pressure increases the slip ratio decreases[2].

$$S = \frac{V_g}{V_f} \tag{8}$$

Relation between dryness fraction (x) and α:

The ratio of mass flow rate of steam from the riser to the total mass flow of water entering the riser is called as the dryness fraction. The dryness fraction (x) and the wetness fraction $(1-x)$ can be written in terms of area, velocity, mass flow, and specific volume as shown in Eqns. (10) and (12), respectively.

dryness fraction,

$$x = \frac{\dot{m}_g}{\dot{m}} = \frac{\rho_g A_g V_g}{\dot{m}} \tag{9}$$

$$x = \frac{A_g V_g}{\dot{m} v_g} \tag{10}$$

wetness fraction,

$$1 - x = \frac{\dot{m}_f}{\dot{m}} = \frac{\rho_f A_f V_f}{\dot{m}} \tag{11}$$

$$1 - x = \frac{A_f V_f}{\dot{m} v_f} \tag{12}$$

Now dividing Eqn. (10) by Eqn. (12) and utilizing Eqns. (7) and (8) one can get Eqn. (16)

$$\frac{x}{1-x} = \frac{A_g V_g}{\dot{m} v_g} \times \frac{\dot{m} v_f}{A_f V_f} = \frac{A_g}{A_f} \times \frac{V_g}{V_f} \times \frac{v_f}{v_g} \tag{13}$$

$$\frac{x}{1-x} = \frac{\alpha}{1-\alpha} \times S \times \frac{v_f}{v_g} \tag{14}$$

$$\alpha = \frac{1}{1 + \frac{1-x}{x} \times S \times \frac{v_f}{v_g}} \tag{15}$$

Let, $= S \times \frac{v_f}{v_g}$, then Eqn. (16) finally transformed into:

$$\alpha = \frac{1}{1 + \frac{1-x}{x} \times \varphi} \tag{16}$$

In Eqn. (16), φ will be a constant because, for a particular boiler pressure the S, v_f and v_g will be constants.

3. Mean density derivation

The mass of mixture in the riser can be written as the sum of vapour and liquid masss, as shown in Eqn. (17).

$$m = m_g + m_f \tag{17}$$

Eqn. (17) can be further be written as Eqn. (18) by using the definition of density.

$$\rho\forall = \rho_g\forall_g + \rho_f\forall_f \tag{18}$$

This can further be written in terms of the density of saturated liquid and vapour, and α (Eqn. 22), as shown below:

$$\rho\forall = \rho_g\forall_g + \rho_f(\forall - \forall_g) \tag{19}$$

$$\rho = \rho_g\frac{\forall_g}{\forall} + \rho_f\left(1 - \frac{\forall_g}{\forall}\right) \tag{20}$$

$$\rho = \rho_g\alpha + \rho_f(1-\alpha) \tag{21}$$

$$\rho = \rho_f - (\rho_f - \rho_g)\alpha \tag{22}$$

Now, the mean density of the mixture in the riser is defined as (refer **Figure 3**):

$$\rho_m = \frac{1}{H}\int_0^H \rho\, dh \tag{23}$$

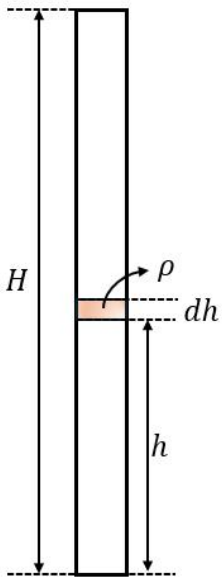

Figure 3. Elemental strip of a two-phase mixture in the riser.

On replacing the expression of ρ from Eqn. 22 into Eqn. 23, and assuming that the density of saturated liquid and vapour does not change in the riser, the Eqn. (25) can be obtained.

$$\rho_m = \frac{1}{H}\int_0^H \left(\rho_f - (\rho_f - \rho_g)\alpha\right) dh \tag{24}$$

$$\rho_m = \rho_f - \frac{1}{H}(\rho_f - \rho_g)\int_0^H \alpha\, dh \tag{25}$$

Now, replacing the value of α from Eqn. 16 into Eqn. 25, one can get Eqn. (26)

$$\rho_m = \rho_f - \left[\frac{(\rho_f - \rho_g)}{H}\right]\int_0^H \frac{dh}{1+\frac{1-x}{x}\varphi} \tag{26}$$

For the sake of simplicity, let say that the integral part be denoted by I, as shown in Eq. (27).

$$I = \int_0^H \frac{dh}{1+\frac{1-x}{x}\varphi} \tag{27}$$

On further simplifying Eqn. (27), one can arrive at Eqn. 31, as shown below:

$$I = \int_0^H \left(\frac{dh}{\varphi+x(1-\varphi)} \frac{(1-\varphi)+\varphi-\varphi}{(1-\varphi)}\right) \tag{28}$$

$$I = \int_0^H \frac{1}{(1-\varphi)} \times \left(1 - \frac{\varphi}{\varphi+x(1-\varphi)}\right) dh \tag{29}$$

$$I = \frac{1}{(1-\varphi)}\int_0^H \left[1 - \frac{\varphi}{\varphi+x(1-\varphi)}\right] dh \tag{30}$$

$$I = \frac{1}{(1-\varphi)}\left[H - \int_0^H \frac{\varphi}{\varphi+x(1-\varphi)} dh\right] \tag{31}$$

Let the term in the integral part be denoted as II, as shown in Eqn. (32).

$$II = \int_0^H \frac{\varphi}{\varphi+x(1-\varphi)} dh \tag{32}$$

Now, to solve Eqn. (32) further, a relation between x and h should be established. Let us assume that the quality of the mixture varies linearly with the riser height i.e. $x \propto h$. By removing the proportionality, Eqn. (33) is obtained, subject to a condition that, when $h = H$, $x = x_o$, where x_o is the quality at the top, which is also called as Top Dryness Fraction (TDF). The value of k then come out as x_o/H.

$$x = kh \tag{33}$$

Let the denominator of Eqn. (32) be represented by t, as shown in Eqn. (34)

$$t = 1 + \frac{1-\varphi}{\varphi}x \tag{34}$$

Which when differentiated (after keeping the value of x) will return:

$$dt = \frac{1-\varphi}{\varphi}\frac{x_o}{H}dh$$

Therefore, term II becomes:

$$II = \frac{\varphi}{1-\varphi}\frac{H}{x_o}\int_1^{1+\frac{1-\varphi}{\varphi}x_o} \frac{dt}{t} \tag{35}$$

Which on further simplification, yields Eqn. (36).

$$II = \frac{\varphi}{1-\varphi}\frac{H}{x_o} \times ln\left[1 + \frac{1-\varphi}{\varphi}x_o\right] \tag{36}$$

For the sake of simplicity, the right-hand side of Eqn. (36) is split into terms III and IV, as shown in Eqns. (37) and (38).

$$III = \frac{\varphi}{1-\varphi}\frac{H}{x_o} \tag{37}$$

$$IV = ln\left[1 + \frac{1-\varphi}{\varphi}x_o\right] \tag{38}$$

Now, writing the TDF in terms of the void fraction at the top (α_o) using Eqn. (16), one can get Eqn. (42).

$$\alpha_o = \frac{1}{1+\frac{1-x_o}{x_o}\times\varphi} \tag{39}$$

$$1 + \frac{1-x_o}{x_o} \times \varphi = \frac{1}{\alpha_o} \tag{40}$$

$$\left(\frac{1}{x_o} - 1\right) = \frac{1-\alpha_o}{\varphi\alpha_o} \tag{41}$$

$$x_o = \frac{\varphi\alpha_o}{1-(1-\varphi)\alpha_o} \tag{42}$$

Therefore, substituting Eqn. (42) into Eqn. (37) and (38), the terms III and IV will be represented in terms of α_o, as shown below:

$$III = \frac{\varphi}{1-\varphi}\frac{H}{x_o} = \frac{\varphi H}{1-\varphi}\frac{(1-(1-\varphi)\alpha_o)}{\varphi\alpha_o} \tag{43}$$

$$III = H\left[\frac{1}{(1-\varphi)\alpha_o} - 1\right] \tag{44}$$

$$IV = ln\left[1 + \frac{1-\varphi}{\varphi}x_o\right] = ln\left[1 + \frac{1-\varphi}{\varphi}\frac{\varphi\alpha_o}{1-(1-\varphi)\alpha_o}\right] \tag{45}$$

$$IV = ln\left[\frac{1}{1-(1-\varphi)\alpha_o}\right] \tag{46}$$

Now, replacing the value of III and IV from Eqns. (44) and (46) into Eqn. (36) the value of the term II will be obtained as:

$$II = H\left[\frac{1}{(1-\varphi)\alpha_o} - 1\right] \times ln\left[\frac{1}{1-(1-\varphi)\alpha_o}\right] \tag{47}$$

Now, replacing the value of II from Eqn. (47) in Eqn. (31), the value of term I will be obtained as:

$$I = \frac{H}{(1-\varphi)}\left[1 - \left[\frac{1}{(1-\varphi)\alpha_o} - 1\right] \times ln\left[\frac{1}{1-(1-\varphi)\alpha_o}\right]\right] \tag{48}$$

Finally, when the value of I is replaced in Eqn. (26), the mean density of the mixture in the riser can be obtained, as shown in Eqn. (49).

$$\rho_m = \rho_f - \left[\frac{(\rho_f - \rho_g)}{(1-\varphi)}\right]\left[1 - \left[\frac{1}{(1-\varphi)\alpha_o} - 1\right] \times ln\left[\frac{1}{1-(1-\varphi)\alpha_o}\right]\right] \tag{49}$$

4. Conclusion

In conclusion, this article has provided a development of the mathematical expression for the mean average mixture density of water in the riser of a subcritical natural circulation boiler. While this expression has been mentioned in various books and literature, a comprehensive explanation of its derivation has been lacking. Through this work, we have aimed to fill this gap by providing a detailed explanation of the origin of this expression.

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Conflict of interest

The authors declare no conflict of interest.

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