

## RESEARCH ARTICLE

# A computational model for the dietary control of insulin, glucose, fatty acid metabolism and type 2 diabetes

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## ABSTRACT

Non-alcoholic fatty liver disease (NAFLD) is a significant global public health issue, closely related to poor dietary habits and excessive energy intake. Type 2 diabetes (T2DM) is closely related to NAFLD. Due to the complex regulation of dietary factors on the interaction between insulin, glucose, and free fatty acids (FFA), existing metabolic models have limitations in characterizing the dynamic response of this system. This paper uses an improved mathematical model to simulate the dynamic effects of different dietary compositions on insulin, glucose, and FFA, the study adopts a delayed feedback mechanism to construct a system of differential equations, which describes the relationship between postprandial insulin secretion and the fluctuations of glucose and FFA. The results show that the model can effectively simulate the fluctuating behavior of metabolic parameters under postprandial conditions, verifying its predictive potential in the study of NAFLD and dietary interventions.

**Keywords:** Non-alcoholic fatty liver disease; free fatty acids; Insulin; Glucose; Dietary; Type 2 diabetes.

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## 1. Introduction

Non-alcoholic fatty liver disease (NAFLD) is a disease marked by the accumulation of fat within the liver. Its pathogenesis is complex, and free fatty acids (FFA) play a crucial role in it. The relationship between type 2 diabetes (T2DM) and NAFLD is complex and bidirectional<sup>[1, 2]</sup>. It is difficult to distinguish whether NAFLD plays a causal role in the development of T2DM or whether it is simply a result of T2DM<sup>[3]</sup>. High FFA levels can lead to metabolic disorders like insulin resistance and hepatocyte injury, which worsen the development of NAFLD<sup>[4, 5]</sup>. Studies have shown that NAFLD is closely related to insulin resistance, which may be the core mechanism leading to reduced insulin levels and increased FFA<sup>[6, 7]</sup>. Insulin resistance may raise FFA levels in the blood. This can lead to fat accumulation and inflammation in the liver, which may then develop into NAFLD. The lipid accumulation and inflammation in NAFLD can be aggravated, making patients with T2DM more susceptible to liver damage<sup>[6, 7]</sup>. Consequently, an in-depth exploration of the association between insulin, FFA, NAFLD and T2DM is highly significant for formulating effective intervention strategies.

Exist mathematical models have been developed to simulate metabolic interactions and provide insights into the regulation of glucose, insulin, and lipid levels in response to dietary inputs.

Classical models, such as those by Sturis et al.<sup>[8]</sup> and Tolić et al.<sup>[9]</sup>, have been instrumental in establishing foundational knowledge about the glucose-insulin feedback system. These models describe core mechanisms, including insulin-stimulated glucose uptake and inhibition of hepatic glucose production. However, their design often limits the exploration of lipid metabolism and postprandial FFA responses, as they focus predominantly on glucose and insulin dynamics, thus excluding important dietary influences. More recent models, König et al.<sup>[10]</sup> and Dalla et al.<sup>[11]</sup> have made significant strides by incorporating multiple compartments to reflect organ-specific metabolic functions, enabling a more realistic simulation of postprandial states. These models, however, frequently overlook the broader systemic effects of diet and the detailed interaction between FFA and glucose metabolism. Mathematical modeling has emerged as a powerful tool to understand metabolic systems by integrating biochemical interactions and simulating metabolic changes under various conditions. Models focusing on insulin-glucose dynamics<sup>[8, 9]</sup> and hepatic lipid metabolism<sup>[12]</sup> provide insight into key regulatory processes. These models help explore how diet and energy balance influence insulin sensitivity, hepatic glucose uptake, and FFA levels in the bloodstream. However, many existing models either simplify these dynamics or overlook the comprehensive effects of dietary inputs on systemic responses.

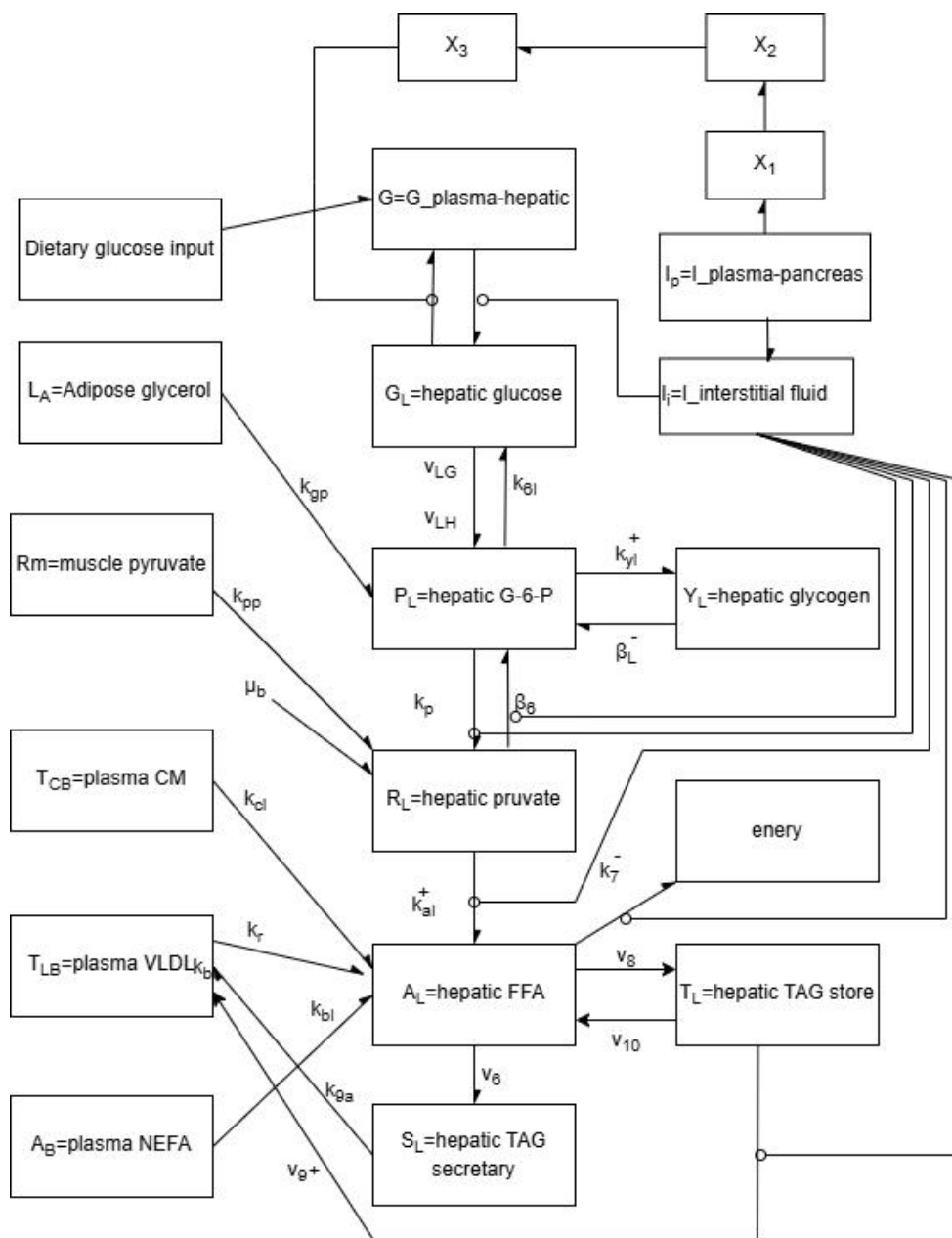
This study aims to develop a mathematical model that encapsulates the interactions between glucose, insulin, and FFA regulated by diet. Building on the glucose-insulin feedback model of Tolić et al.<sup>[9]</sup> and Pratt et al.<sup>[12]</sup> framework for hepatic lipid metabolism, this model simulates the metabolic response to mixed meals under fed and fasted states. We hypothesize that by incorporating dietary variables, this model will more accurately reflect postprandial oscillations in insulin and FFA levels and can thus serve as a predictive tool for metabolic studies, dietary intervention planning, and clinical applications.

## 2. Materials and methods

This model is founded on several hypotheses backed by the literature<sup>[8, 9, 12, 13]</sup>. Nevertheless, to prevent this stage of model development from becoming too onerous, some details of the process are omitted. Sturis and Tolić<sup>[8, 9]</sup> proposes a systematic insulin-glucose model which served as a basis for the glucose and insulin equations. Similarly, Pratt and Kosic<sup>[12, 13]</sup> described the (fasting) FFA kinetics are described by kinetic equations. The following insulin-glucose feedback loops are included in the model: glucose stimulates pancreatic insulin secretion, insulin stimulates glucose uptake and inhibits hepatic glucose production and glucose enhances its own uptake. The system contains two significant delays. One delay is related to the fact that the physiological action of insulin on the utilization of glucose is correlated with the concentration of insulin in a slowly equilibrating intercellular compartment rather than with the concentration of insulin in the plasma. The other delay is associated with the time lag between the appearance of insulin in the plasma and its inhibitory effect on the hepatic glucose production. The insulin-glucose model has three main variables: the amount of glucose in the plasma and intercellular space,  $G$ , the amount of insulin in the plasma,  $I_p$ , and the amount of insulin in the intercellular space,  $I_i$ . In addition, there are three variables,  $x_1$ ,  $x_2$ , and  $x_3$ , that represent the above-mentioned delay between insulin in plasma and its effect on the hepatic glucose production.  $V_p$  is the distribution volume for insulin in plasma, and  $V_i$  the effective volume of the intercellular space. Insulin degradation is assumed to be exponential, with time constant  $t_p$  for insulin in plasma and  $t_i$  for insulin in the intercellular space. Assume that the delay is of third order and the total time is  $t_d$ .  $G_{in}$  with the glucose infusion rate of  $216 \text{ mg min}^{-1}$ . Hepatic glucose,  $G_L$ ; Hepatic glycogen,  $Y_L$ ; Hepatic-6-phosphahte,  $P_L$ ; Free fatty acids in liver,  $R_L$ ; Triacylglycerol storage pool in liver,  $A_L$ ; TAG storage pool  $T_L$ ; TAG secretary pool in liver,  $S_L$ ; Adipose glycerol,  $L_A$ ; Muscle pyruvate,  $R_m$ ; Plasma exogenous CM TAG,  $T_{CB}$ ; Plasma endogenous LP TAG,  $T_{LB}$ . The overall scheme of reactions and transport is illustrated in **Figure 1**.

The models were represented in SBML<sup>[14]</sup>, which is a standard for biochemical networks, using the Python tool SBML shorthand<sup>[15]</sup>. Model diagrams were constructed in CellDesigner<sup>[16]</sup> in accordance with the Systems Biology Graphical Notation (SBGN)<sup>[17]</sup>. The SBML code was deposited in the BioModels database and assigned the identifier BIOMD0000000372<sup>[9]</sup> and BIOMD0000000382<sup>[8]</sup>, but Pratt's model<sup>[12]</sup> and Kotic's model<sup>[13]</sup> not in BioModels, so we reconstruct this model through the details of the paper and omit the unnecessary parts, such as muscle, adipose tissue.

In this paper, an analysis of the mathematical model of hepatic lipid metabolism was carried out. The model in the form of 18 differential kinetic equations (Supplementary Material **Table 1**) including 64 parameters (Supplementary Material **Table 2**) describes the metabolic response of the organism to meals with different proportions of macronutrients with a special emphasis on FFA. Pratt et al.<sup>[12]</sup> propose the macronutrient metabolism pathway. The model includes three compartments: liver, blood plasma and pancreas.



**Figure 1.** Dietary control of insulin, glucose, and fatty acid model<sup>[8, 9, 12]</sup>

Model diagram: at the top we show dietary input of glucose, on the right and center are hepatic components, on the upper right are pancreas, on the left are plasma compartments, In addition to the variable names and descriptions, we include a parameter associated with each flux, plus and minus superscripts represent insulin stimulated and insulin inhibited pathways (A description of each species is in the supplement).

### 3. Results

#### 3.1. Simulation of insulin and hepatic glucose level in normal condition

The oscillation amplitude of insulin secretion increased with the increase of glucose infusion rate, which was divided into two parts, one was intravenous injection, the other was oral glucose absorption, and the total absorbed glucose was  $214.6 \text{ mg min}^{-1}$ . When the oral glucose is set at  $115.28 \text{ mg min}^{-1}$  and the intravenous infusion is set at  $99.32 \text{ mg min}^{-1}$ , if the intravenous and oral glucose are continuously given, hepatic glucose and insulin show periodic oscillations. (Figure 2)

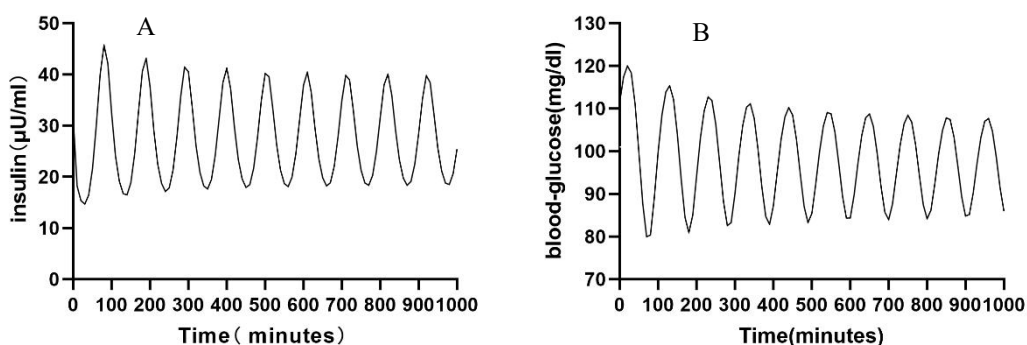
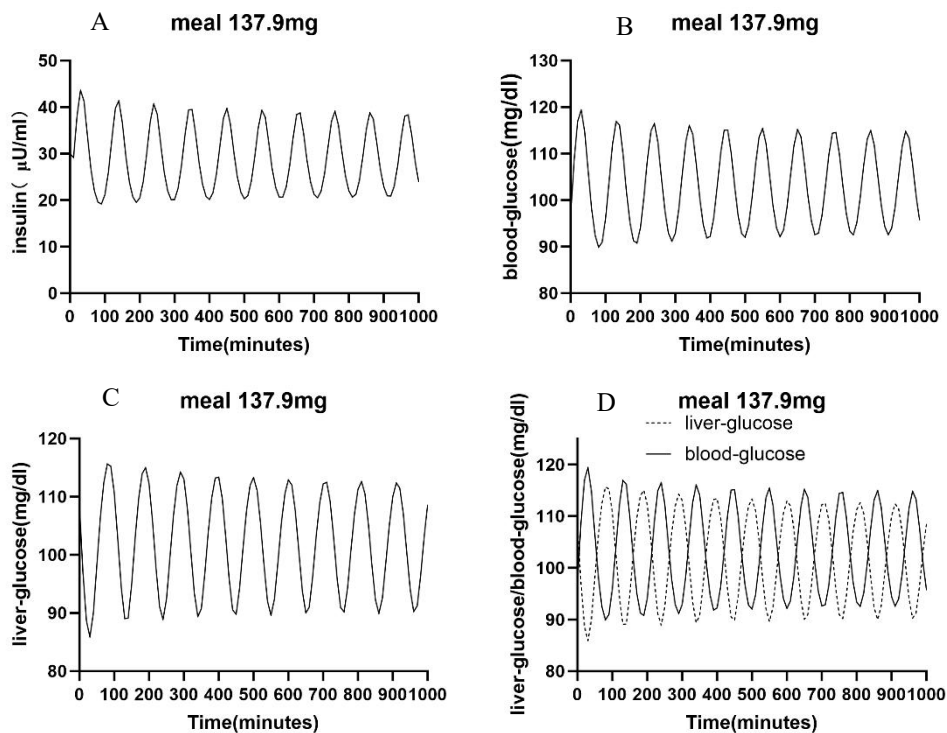
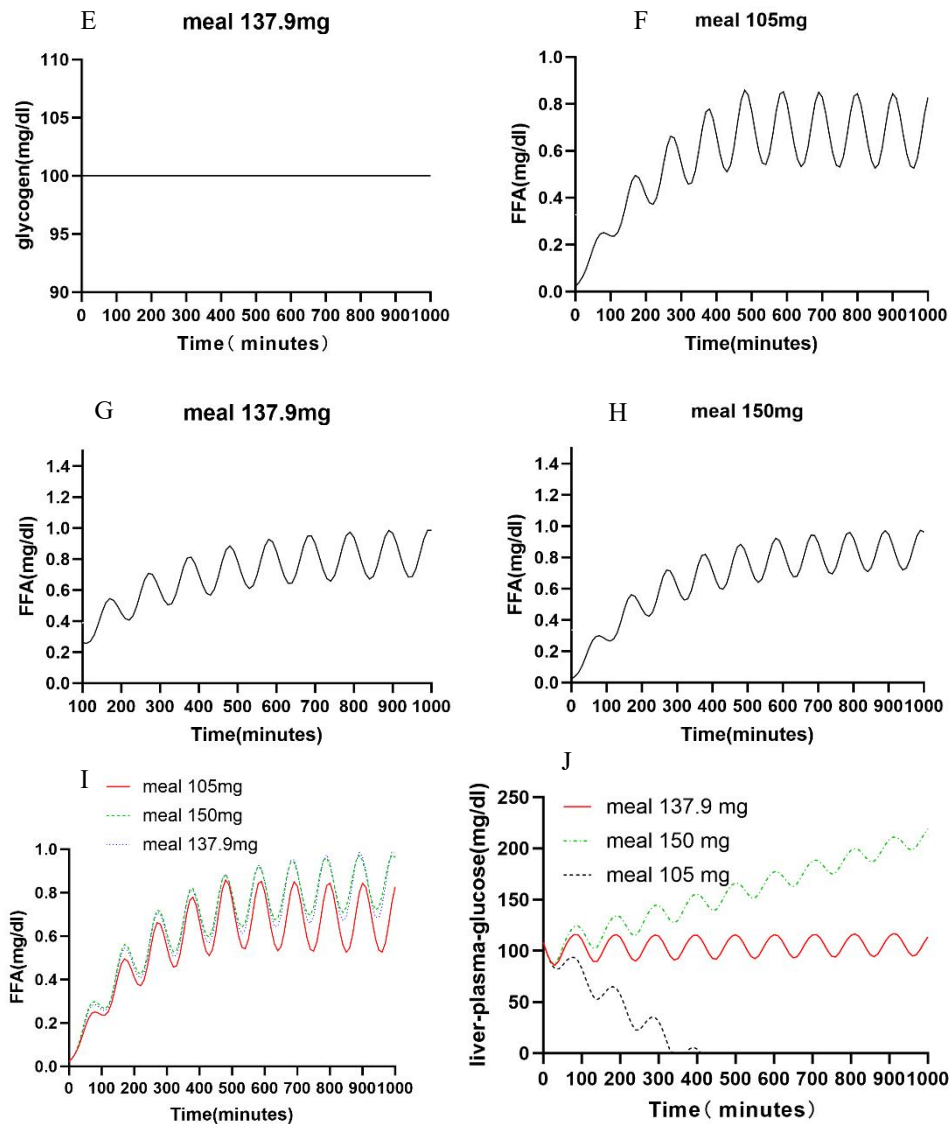


Figure 2. The time evolution of the insulin (A) and blood glucose (B) concentration.

#### 3.2. Hepatic glycogen, insulin, plasma glucose and FFA at different diet intake





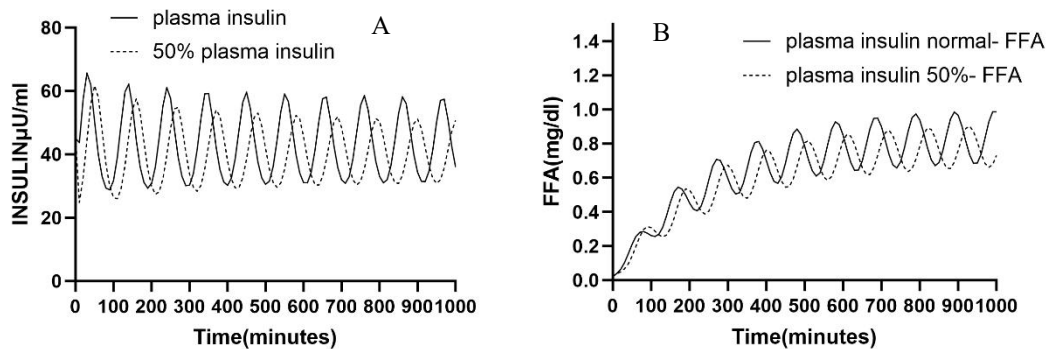
**Figure 3.** A predictive model of hepatic glycogen, insulin, glucose and FFA metabolism.

Compared with **Figure 3** (A,B) and **Figure 2** (A,B), the amplitude of insulin and blood glucose did not change substantially, indicating that the fitting of the two models did not affect the original model. When the intake was 137.9 mg glucose (higher than the equilibrium value of the model alone), (**Figure 3A**) Liver plasma insulin feedback, (**Figure 3B**) glucose in blood and (**Figure 3C**) the glucose in liver changed. Simultaneous contrast of hepatic glucose and blood glucose (**Figure 3D**); Changes in hepatic glycogen (**Figure 3E**). The changes of FFA when the intake was 105 mg (**Figure 3F**), 137.9 mg (**Figure 3G**), and 150mg (**Figure 3H**) respectively. (**Figure 3I**) Simultaneous comparison of fatty acid changes when meal intake was 105 mg (red line), 137.9 mg (blue dashes) and 150 mg (green dashes), respectively. (**Figure 3J**) Simultaneous comparison of liver plasma glucose changes when meal intake was 105 mg (black line), 137.9 mg (red line) and 150 mg (green dashes), respectively

**Figure 3B** shows that when the food intake is 137.9 mg containing glucose, liver blood glucose fluctuates between 120-80 mg, which is consistent with the physiological phenomenon of mammals. **Figure 3C** shows that hepatic glucose also fluctuates between 120 mg-80 mg. In **Figure 3D**, glucose in the blood and glucose in the liver oscillate opposite, that is, glucose in the blood is at the highest amplitude, but glucose in the liver is at the trough; In contrast, while glucose in the blood troughs, glucose in the liver peaks and oscillates consistently with each other. In **Figure 3E**, the hepatic glycogen content remained constant at 100 g, neither increasing nor decreasing with the increase of food intake, which was consistent with

mammalian physiological phenomena. As shown in **Figure 3F**, when the food intake was 105mg and glucose content, the free fatty acids increased rapidly and maintained a fixed level of oscillation; When the food intake increased to 137.9 mg (**Figure 3G**), the FFA also increased, and finally fluctuated in a fixed range. When the meal intake was increased to 150 mg (**Figure 3H**) containing glucose, the free fatty acid shock gradually disappeared. **Figure 3I** shows the simultaneous comparison of FFA content changes when the meal intake was 105 mg (low level), 137.9 mg equilibrium) and 150 mg (high-level). **Figure 3J** shows the simultaneous comparison of glucose content changes when the meal intake was 105 mg (glucose decline), 137.9 mg (glucose equilibrium) and 150 mg (glucose rise).

### 3.3. Simulation of insulin and FFA levels in diabetic condition



**Figure 4.** Simulation of normal insulin secretion and reduce secretion by 50%.

In the later stages of diabetes, insulin secretion usually decreases significantly and may be only about 50% of the level in normal healthy human<sup>[18]</sup>. In the model, we simulated 50% of normal insulin production (**Figure 4A**) and explored the relationship between insulin and FFA (**Figure 4B**). The results showed that when insulin production decreased, FFA production also decreased.

## 4. Discussion

This study model provides a simplified framework for investigating the effects of diet on postprandial blood glucose metabolism and lipid metabolism in the liver, which is suitable for the study of metabolic disorders like T2DM mellitus and NAFLD. This section will discuss the advantages and disadvantages of this model, such as the benefit of delayed insulin secretion, the lack of  $\beta$ -cell mass, limitations, and directions for improvement.

The design of this model offers significant application potential for metabolic studies focusing on the response of glucose and lipids to different dietary components after meals. Similar models have been applied to model the metabolic dynamics of T2DM and NAFLD, in which fluctuations in glucose and FFA play a crucial role in disease progression.<sup>[19-21]</sup> The model is aligned with contemporary research trends by incorporating dietary responses, with an emphasis on dietary adjustments for controlling blood glucose and lipids. These applications also demonstrate the usefulness of the model in simulating the metabolic responses to different dietary components in the context of metabolic syndrome (e.g., T2DM, NAFLD, etc.), and offer insights into dietary intervention strategies for the management of metabolic syndrome. The combination of dietary input is the highlight of this model, which can simulate the postprandial response of blood glucose and hepatic glucose in both feeding and fasting states, overcoming the limitations of the model that focuses only on intravenous glucose infusion, and providing a more realistic description of glucose and lipid metabolism. However, it neglects glycogen intermediates such as glucose-1-phosphate, uridine diphosphate glucose, inorganic phosphate, and pyrophosphate, which play a vital role in the conversion between blood glucose and glycogen and have a considerable impact on metabolic pathways. Nevertheless, the model

disregards them.<sup>[22]</sup> In addition, simplifying hepatic lipid metabolism, such as ignoring protein interactions such as skeletal muscle, adipose tissue, and plasma variables, may limit the application of this model in high-fat diet studies<sup>[5]</sup>. Another deficiency is the lack of beta cell mass production, which is not a good description of beta cell changes under hyperglycemia. Classic  $\beta$  cell mass model is Topp et al.<sup>[23]</sup>, mild hyperglycemia has a negative feedback mechanism with  $\beta$  cell mass, while extreme hyperglycemia has a positive feedback mechanism with  $\beta$  cell mass. Another model development  $\beta$  cell mass model of diabetes in fa/fa rats in 2007<sup>[24]</sup>, show that both excessive insulin resistance and insufficient adaptation contribute to the initiation of hyperglycemia<sup>[24]</sup>. Current models view beta cell mass as static, restricting its application in long-term metabolic studies. Not modeling the changes in beta cell populations adapting to chronic insulin needs over time can result in inaccurate predictions of persistent hyperglycemia<sup>[25]</sup>. For example, in people with long-term diabetes, beta cell mass may gradually decrease. Models that do not reflect this change will overestimate the amount of insulin secreted, resulting in inaccurate predictions of blood glucose levels.

The delayed insulin secretion is a very important feedback link in the model of insulin metabolism. If there were no delay, insulin would not oscillate. Therefore, the analysis of insulin delay is the key to accurately simulate the metabolic response. The model incorporates the insulin secretion delay, so it can capture the oscillation behavior of insulin-glucose in the feedback loop. Such as those described by recent studies<sup>[26]</sup>, simulate time lags between insulin secretion and its physiological effects, crucial for maintaining metabolic stability. For instance, insulin delays allow for more precise regulation of blood glucose following dietary intake, as delayed secretion can prevent overcompensation that leads to hypoglycemia. These delays also reflect real-life physiological conditions where insulin effects on glucose metabolism are not immediate. In comparison to models without insulin secretion delays<sup>[27]</sup>, such as purely linear feedback models, our model provides an advantage by offering a closer approximation of physiological insulin responses. Including delays enables simulations of postprandial glucose and FFA oscillations, reflecting the temporal dynamics essential for metabolic stability. This is particularly useful for studies focused on oscillatory regulation in metabolic systems, as disruptions in delay timings have been linked to insulin resistance and glucose intolerance<sup>[28]</sup>. The delay mechanism in this model also allows for exploring potential therapeutic approaches that target insulin secretion timing, highlighting the role of temporal regulation in managing metabolic disorders.

In the later stage of diabetes, especially in patients with type 2 diabetes, as the disease progresses, the function of  $\beta$  cells gradually declines and the ability to secrete insulin is significantly reduced. Studies have shown that patients with type 2 diabetes secrete 50% to 70% less insulin than normal people<sup>[29, 30]</sup>, and many diabetics have insulin resistance before the insulin level is lower in the later stage<sup>[31, 32]</sup>. This resistance leads to increased release of FFA. This, in turn, leads to the accumulation of fat in the liver. For example, the liver uses excessive free fatty acids to synthesize triglycerides and store them to form fatty liver<sup>[33, 34]</sup>. In the later stages of diabetes, however, fatty liver disease can be greatly improved through weight loss promotion or the direct treatment of liver fat accumulation<sup>[35]</sup>. This model can well reproduce the corresponding changes in blood glucose, hepatic glucose and FFA when insulin secretion is reduced, which is consistent with the reality. When insulin production is 50% of normal, it can be found that blood glucose and hepatic glucose increase significantly, which affects the content of free fatty acids. Nicely reproduced that T2DM patients are more likely to develop fatty liver disease, providing insights into the relationship between T2DM and FFA.

Future studies of this model should focus on exploring the effects of glucose changes on pancreatic  $\beta$  cells, including their generation and death, and the effect of circadian genes on blood glucose balance and hepatic lipid metabolism. Moreover, this can be achieved by combining islet  $\beta$  cell mass changes, hepatic lipid metabolism intermediates, and circadian factors of fatty acid levels, like cortisol and melatonin.

## 5. Conclusion

In conclusion, this insulin-glucose-FFA model provides a versatile tool for exploring dietary impacts on postprandial metabolism across various scenarios. Future iterations that integrate adaptive  $\beta$ -cell responses and insulin secretion delays may strengthen its relevance, offering a valuable resource for both research and clinical applications in metabolic disease management.

## Conflict of interest

The authors declare no conflict of interest.

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## Supplementary Material

**Table 1.** The equations describing the dynamics of the Insulin, Glucose, and FFA model

Number	Variable	Variable	Balance	Initial Conditions
1	the amount of insulin in the plasma	$I_p$	$\frac{dI_p}{dt} = f_1(G) - E \left( \frac{I_p}{V_p} - \frac{I_i}{V_i} \right) - \frac{I_p}{t_p}$	90.0 mmol/L
2	In addition, there are three variables	$I_i$	$\frac{dI_i}{dt} = E * \left( \frac{I_p}{V_p} - \frac{I_i}{V_i} \right) - \frac{I_i}{t_i}$	138 mmol/L
3	glucose in the plasma and intercellular space	$G$	$\frac{dG}{dt} = G_{in} - f_2(G) - f_3(G) * f_4(I_i) + f_5(x_3)$	13000 mmol/L
3(1)	glucose in the plasma and intercellular space	$G$	$\frac{dG}{dt} = G_{in} - f_2(G) - \frac{1}{5} f_3(G) * f_4(I_i) - \frac{4}{5} f_3(G) * f_4(I_i) + f_5(x_3)$	13000 mmol/L
4	represent the above-mentioned	$X_1$	$\frac{dx_1}{dt} = \frac{3}{t_d} * (I_p - x_1)$	70 mmol/L
5	delay between insulin in plasma and its effect on the hepatic glucose production	$X_2$	$\frac{dx_2}{dt} = \frac{3}{t_d} * (x_1 - x_2)$	70 mmol/L
6		$X_3$	$\frac{dx_3}{dt} = \frac{3}{t_d} * (x_2 - x_3)$	70 mmol/L
7	The pancreatic insulin production controlled by the glucose concentration	$f_1(G)$	$f_1(G) = \frac{R_m}{1 + \exp((C_1 - G/V_g)/a_1)}$	-
8	Insulin-independent glucose utilization (glucose uptake by the brain and nerve cells)	$f_2(G)$	$f_2(G) = U_b(1 - \exp(-G/(C_2 * V_g)))$	-
9	The glucose dependent term in the function describing glucose utilization is assumed to be which agrees with experimental result	$f_3(G)$	$f_3(G) = \frac{G}{C_3 * V_g}$	-
10	The insulin dependent term	$f_4(I_i)$	$f_4(I_i) = U_0 + \frac{U_m - U_0}{1 + \exp(-\beta \ln(I_i/C_4(1/V_i + 1/Et_i)))}$	-
11	The influence of insulin on the hepatic glucose production	$f_5(x_3)$	$f_5(x_3) = \frac{R_g}{1 + \exp(a * (x_3/V_p - C_5))}$	-
12	liver glucose	$G_L$	$\alpha_L \frac{dG_L}{dt} = S_G(t) - k_{gl}G_L + k_{gl2}G_B - \frac{v_{LG}G_L}{k_{LG} + G_L} - \frac{v_{LH}G_L}{k_{LH} + G_L} \left( \frac{1}{1 + k_{rep}P_L} \right) + k_{6l}P_L$	8 mmol/L
13	liver glycogen	$Y_L$	$\alpha_L \frac{dY_L}{dt} = \frac{1}{2} k_{yl}IP_L \left( 1 + \tanh \left( \frac{I_{max} - Y_L}{C_0} \right) \right) - \frac{\beta_L}{1 + k_{di}I} \left( \frac{Y_L}{Y_L + y_0} \right)$	50 mmol/l

**Table 1. (Continued)**

14	liver glucose-6-phosphate	$P_L$	$\alpha_L \frac{dP_L}{dt} = -\frac{1}{2} k_{yI} I P_L \left( 1 + \tanh \left( \frac{I_{max} - Y_L}{C_0} \right) \right) + \frac{\beta_L}{1 + k_{dI} I} \left( \frac{Y_L}{Y_L + y_0} \right) + \frac{\beta_6 R_L}{1 + k_{p6} I} + \frac{v_{LG} G_L}{k_{LG} + G_L}$ $+ \frac{v_{LH} G_L}{k_{LG} + G_L} \left( \frac{1}{1 + k_{rep} P_L} \right) - k_p I P_L - k_{6I} P_L + k_{gp} L_A$	2.06 mmol/l
15	free fatty acids in liver	$R_L$	$\alpha_L \frac{dR_L}{dt} = k_{pP} R_M + k_p I P_L - \frac{\beta_6 R_L}{1 + k_{p6} I} - k_{aI} I R_L + \mu_B$	0.37 mmol/L
16	triacylglycerol storage pool in liver	$A_L$	$\alpha_L \frac{dA_L}{dt} = 3k_{cl} T_{CB} + k_{bl} A_{NB} + 3k_r T_{LB} + k_{aI} I R_L + \frac{3v_{10} T_L}{k_{10} + T_L} + \frac{3v_6 A_L}{k_6 + A_L} - \frac{3v_8 A_L}{k_8 + A_L} - \frac{k_7 A_L}{1 + k_5 I}$	0.57 mmol/L
17	TAG storage pool	$T_L$	$\frac{dT_L}{dt} = \frac{v_8 A_L}{k_8 + A_L} - F(I) \frac{v_9 T_L}{k_9 + T_L} - \frac{v_{10} T_L}{k_{10} + T_L}$	40 mmol/L
18	TAG secretory pool in liver	$S_L$	$\frac{dS_L}{dt} = \frac{3v_6 A_L}{k_6 + A_L} - k_{9a} S_L$	0.0149 mmol/l

**Table 2.** List of dynamics of the Insulin, Glucose, and FFA model parameters [8, 9, 12, 13]

Number	name	value	Description
1	$\alpha_A$	15.6L	adipose tissue volume
2	$\alpha_L$	1.60 L	liver tissue volume
3	$\beta_6$	31.6L/min	rate of liver de novo lipogenesis from pyruvate
4	$\beta_L$	12L/min	liver glycogenolysis
5	$\mu_1$	0.588 mmol/min	Plasma glucose usage
6	$C_0$	0.1mmol/L	small parameters
7	$k_5$	0.05/mmol	flux control coefficient for insulin inhibition of free fatty acid oxidation
8	$k_{61}$	4L/min	liver glucose dephosphorylation rate
9	$k_6$	0.3mmol/L	affinity for very low-density lipoprotein 2 triglyceride secretion through secretory pathway
10	$k_7$	0.759L/min	maximum rate of free fatty acid oxidation
11	$k_8$	0.625mmol/L	affinity for esterification of free fatty acids to triglycerides
12	$k_9$	43.583mmol/L	affinity of additional bulk lipidation
13	$k_{9a}$	1L/min	release of very low-density lipoproteins from secretory pathway
14	$k_{10}$	0.625mmol/L	affinity for hydrolysis of triglycerides to secretory pool
15	$k_{12}$	0.2	increased fraction of very low-density lipoprotein 1 secretion by insulin
16	$k_{13}$	15mmol/L	rate at which insulin modifies the fraction of very low-density lipoprotein 1 to very low-density lipoprotein 2 secretion
17	$k_{14}$	0.6	basal very low-density lipoprotein 1 secretion fraction

**Table 2. (Continued)**

18	$k_{22}$	48mmol/min	excess insulin secretion rate due to glucose stimulation
19	$k_{al}$	0.00002L <sup>2</sup> /mmol min	pyruvate to acetyl coenzyme A conversion rate
20	$k_{ba}$	0.0104L/min	adipose uptake of endogenous lipoprotein triglycerides
21	$k_{bl}$	0.156L/min	liver uptake of plasma non-esterified fatty acids
22	$k_{cl}$	0.0075L/min	liver free fatty acid uptake of chylomicron triglycerides
23	$k_d$	$1.733 \times 10^8$ L/mmol	insulin degradation rate
24	$k_{dl}$	$3.5 \times 10^8$ mmol/L	liver glycogenolysis; insulin-inhibited rate
25	$k_{gp}$	0.311L/min	glucose-6-phospahte uptake from adipose glycerol
26	$k_{LG}$	0.0115mmol/L	Michaelis - Menten constant of glucokinase in liver
27	$k_{LH}$	0.0115mmol/L	Michaelis - Menten constant of glucokinase in liver
28	$k_P$	$1.41 \times 10^7$ mmol/L	rate of insulin-mediated glucose-6-phoshate to pyruvate
29	$k_{PP}$	0.5	rate of muscele pyruvate transport to liver
30	$k_{p6}$	$1.93 \times 10^8$ L <sup>2</sup> /mmol min	constant of pyruvate conversion to glucose-6-phosphate
31	$k_{yl}$	$1.28 \times 10^6$ L <sup>2</sup> mmol <sup>-1</sup> min <sup>-1</sup>	rate of the glycogen synthesis stimulated by insulin
32	$k_r$	0.00058mmol	rate of endogenously derived lipoprotein triglycerides by liver as free fatty acids
33	$k_{rep}$	2.98mmol/L	glucose-6-phospahte inhibition constant of hexokinase in muscele
34	$k_t$	0.00348mmol/L	uptake rate of plasma endogenous triglycerides into muscle free fatty acids
35	$l_{max}$	400mmol	maximum glycogen store of liver
36	$v_6$	0.6mmol/L	rate of glycogen transport
37	$v_8$	0.333mmol/min	rate of glycogen transport
38	$v_9$	0.6mmol/L	rate of triglyceride release into plasma
39	$v_{10}$	0.1mmol/min	rate of triglyceride storage conversion to free fatty acids
40	$v_{12}$	40mmol L <sup>-1</sup>	constant in triglyceride release into plasma
41	$v_{LG}$	14.3mmol/min	maximum rate of glucokinase in liver
42	$v_{LH}$	5.57mmol/min	maximum rate of hexokinase in liver
43	$y_0$	0.1mmol/min	range of liver glycogen concentration over which the release drops to zero)
44	$V_p$	3L	the distribution volume for insulin in plasma
45	$V_i$	11L	the effective volume of the intercellular space
46	$V_g$	10L	-

**Table 2. (Continued)**

47	E	0.2L min <sup>-1</sup>	transfer rate E between insulin between plasma and intercellular space
48	t <sub>p</sub>	6L min	Insulin degradation is assumed to be exponential , with time constant t <sub>p</sub> for insulin in plasma
49	t <sub>i</sub>	100 L min	Insulin degradation is assumed to be exponential , with time constant t <sub>i</sub> for insulin in the intercellular space
50	t <sub>d</sub>	36 L min	This delay is assumed to be of third order with a total time t <sub>d</sub>
51	R <sub>m</sub>	210 mU min <sup>-1</sup>	-
52	a <sub>1</sub>	300 mg L <sup>-1</sup>	-
53	C <sub>1</sub>	2000 mg L <sup>-1</sup>	-
54	U <sub>b</sub>	72 mg min <sup>-1</sup>	-
55	C <sub>2</sub>	144 mg L <sup>-1</sup>	-
56	C <sub>3</sub>	1000 mg L <sup>-1</sup>	-
57	U <sub>0</sub>	40 mg min <sup>-1</sup>	-
58	U <sub>m</sub> (	940 mg min <sup>-1</sup>	-
59	β	1.77	-
60	C <sub>4</sub>	80 mU I <sup>-1</sup>	-
61	R <sub>g</sub>	180 mg min <sup>-1</sup>	-
62	α	0.29mU I <sup>-1</sup>	-
63	C <sub>5</sub>	26 mU I <sup>-1</sup>	-
64	G <sub>in</sub>	216mg min <sup>-1</sup>	glucose is supplied to the plasma at an exogenously controlled rate

The above glucose-insulin model is based on the mathematical model of blood glucose balance established by Tolic and Sturis team in 2000, and we have made the following adjustments:

The links of glucose generation and glycogen synthesis in the liver were re-established, and gluconeogenesis and glycolysis occurred in the liver. For the sake of simplification of the model, only the links of glucose conversion into glycogen in the liver were concerned, and other links were ignored. Moreover, according to the study<sup>[10]</sup>, the conversion ratio of glycogen to grape in the liver was 80%, that is, the proportion of glucose consumption accounted for 20%, so the equation 3 was changed to 3(1).

$$\frac{dG}{dt} = G_{in} - f_2(G) - \frac{1}{5}f_3(G) * f_4(I_i) - \frac{4}{5}f_3(G) * f_4(I_i) + f_5(x_3)$$

$\frac{1}{5}f_3(G) * f_4(I_i)$  represents the part of glucose that is used in response to insulin,  $\frac{4}{5}f_3(G) * f_4(I_i)$  represents the part of the liver where glucose is converted into glycogen. Considering that the part of glucose absorption includes the part of food digestion and absorption, and the glucose converted by food digestion and absorption as set in the paper, we set 125 mg min<sup>-1</sup>, and the glucose infusion rate of G<sub>in</sub> in the previous equation was 216mg min<sup>-1</sup>, G<sub>in</sub> was split into two parts. One portion represents 125 mg min<sup>-1</sup> for food digestion and absorption, and the other 95mg min<sup>-1</sup> represents intravenous infusion of partial glucose.